

# Sudakov Expansions and Top Quark Physics at LHC

Matteo Beccaria

*Dipartimento di Fisica and INFN, Via Arnesano Ex Coll. Fiorini, 73100 Lecce, ITALY*

**Abstract.** We review some peculiar features of Sudakov expansions in the calculation of electroweak radiative corrections in the MSSM at high energy. We give specific examples and consider in particular the process  $bg \rightarrow tW$  of single top quark production relevant for the top quark physics programme at LHC.

**Keywords:** Minimal Supersymmetric Standard Model, Electroweak Radiative Corrections

**PACS:** 12.15.Lk, 12.60.Jv, 13.75.Cs

## INTRODUCTION

The analysis of physical processes in the MSSM is complicated by the large number of parameters of the model. Also, radiative correction typically involve a large subset of them making apparently quite difficult to isolate specific physical effects. However, there is a way out to simplify the situation at the price of some controlled approximation. Here we shall be concerned with the electroweak sector of the MSSM. The planned future collider experiments (LHC, ILC) will be characterized by large invariant masses and at very high energy it is known that radiative corrections simplify and become smooth, at least beyond production thresholds. In this regime they can be described by rather simple asymptotic expansions. These are basically power series in the logarithm of the center of mass energy or typical large invariant mass. This logarithmic approximation is called a Sudakov expansion and requires light SUSY scenarios with not too heavy sparticles in order to be accurate [1]. Apart from this basic requirement, it compactly expresses the relevant radiative corrections with a very minimal set of parameters [2]. The deep reason for this important feature is that at high energy soft SUSY breaking operators are suppressed for dimensional reasons and disappear at the leading orders in the expansion.

The origin of large logarithms in the physical amplitudes is nowadays quite clear [3]. There are two different kinds of contributions coming respectively from short and long distance and therefore of ultraviolet or infrared nature. The UV logarithms are governed by standard Renormalization Group equations. They typically arise in gauge boson self energy corrections or in essentially gaugeless sectors of the MSSM, like the important Yukawa one. The IR logarithms are manifestations of mass singularities. They can arise in diagrams with exchange of gauge bosons. In the calculation of exclusive processes the mass of massive gauge bosons plays the role of an infrared regulator and gives rise to en-

ergy growing contributions of logarithmic type. For instance, at one loop the leading terms are proportional to  $\alpha \log^2(s/M^2)$  where  $\alpha = e^2/4\pi$ ,  $\sqrt{s}$  is the center of mass energy and  $M$  is a process dependent scale. The unusual counting of two logs per loop is a typical remnant of the IR origin of these contributions.

In the next Section, we shall discuss briefly the classification of the Sudakov logarithms. This will provide the necessary tools to discuss the corrections to single top quark production processes at LHC, as an interesting application.

## SOME CLASSIFICATION OF SUDAKOV LOGARITHMS

The electroweak Sudakov logarithms in a  $2 \rightarrow 2$  process at one loop can be classified according to the following three main categories. We closely follow the notation of [3] and refer to this paper for minor or notational details concerning the next paragraphs.

*a) UV Logarithms from gauge boson self energies.* If  $F^{Born}$  is the amplitude under study, at one loop we get from coupling constants renormalization, the following correction term

$$F^{RG} = -\frac{1}{4\pi^2} \left( g^4 \beta \frac{\partial F^{Born}}{\partial g^2} + g'^4 \beta' \frac{\partial F^{Born}}{\partial g'^2} \right) \log \frac{s}{\mu^2}$$

and of course  $\beta, \beta'$  depend on the model (SM, MSSM, Split SUSY, etc.)

*b) Universal Logarithms.* These are terms independent on the scattering angles. They can be computed by analyzing the various external lines associated to initial or final states one after the other. Each line gets a correction factor that we now discuss. The general form of the

correction is ( $p$  = scalar, fermion or vector)

$$\frac{\alpha}{\pi} c_p^{\text{gauge}} \left( n \log \frac{s}{M_V^2} - \log^2 \frac{s}{M_V^2} \right) + \frac{\alpha}{\pi} c_p^{\text{Yukawa}} \log \frac{s}{M'^2}$$

where  $V = \gamma, Z, W^\pm$ . The linear logarithm scale is a choice to be fixed at NNLO.

In each model and for each kind of external line (initial or final) we can list the various coefficients ( $n$ ,  $c_p$ ,  $c_p^{\text{Yukawa}}$ ) and write immediately the universal corrections at one loop and next to leading logarithmic order.

c) *Angular dependent logarithms.* The final type of logarithms is called angular dependent since depends on the scattering angle. These are terms of the form

$$\frac{\alpha}{\pi} \log \frac{s}{M^2} \log \frac{1 \pm \cos \vartheta}{2},$$

where  $\vartheta$  is the scattering angle. These contributions arise from Standard Model box diagrams and do not receive SUSY additional terms.

### An example: correction to fermion or sfermion external lines

To give an explicit example of the above mentioned corrections, we now consider the specific case of external lines associated to a fermion (lepton or quark) or a sfermion (slepton or squark). The universal logarithms are

$$\frac{\alpha}{\pi} c_f^{\text{gauge}} \left( n \log \frac{s}{M_V^2} - \log^2 \frac{s}{M_V^2} \right) + \frac{\alpha}{\pi} c_f^{\text{Yukawa}} \log \frac{s}{m_t^2}$$

where  $m_t$  is the top quark mass. The gauge part reflects the  $SU(2) \times U(1)$  structure. With standard notation, it reads

$$\begin{aligned} c_f^{\text{gauge}} &= \frac{1}{8} \left[ \frac{I_f(I_f+1)}{s_W^2} + \frac{Y_f^2}{4c_W^2} \right] \\ Y_f &= 2(Q_f - I_f^3) \end{aligned}$$

with  $n = 3$  in the Standard Model and  $n = 2$  in the MSSM. The Yukawa part is present only for heavy quarks top and bottom and reads (the upper line refers to the Standard Model, the lower to the MSSM)

$$c_{b_L} = c_{t_L} = \begin{cases} -\frac{1}{32s_W^2} \left( \frac{m_t^2}{M_W^2} + \frac{m_b^2}{M_W^2} \right) \\ -\frac{1}{16s_W^2} \left( \frac{m_t^2}{M_W^2} \frac{1}{\sin^2 \beta} + \frac{m_b^2}{M_W^2} \frac{1}{\cos^2 \beta} \right) \end{cases}$$

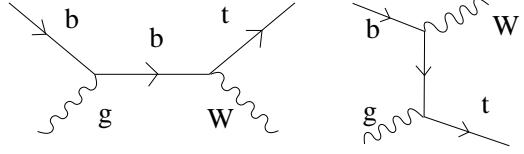


FIGURE 1.  $bg \rightarrow tW$  at tree level.

$$c_{b_R} = \begin{cases} -\frac{1}{16s_W^2} \frac{m_b^2}{M_W^2} \\ -\frac{1}{8s_W^2} \frac{m_b^2}{M_W^2} \frac{1}{\cos^2 \beta} \end{cases} \quad c_{t_R} = \begin{cases} -\frac{1}{16s_W^2} \frac{m_t^2}{M_W^2} \\ -\frac{1}{8s_W^2} \frac{m_t^2}{M_W^2} \frac{1}{\sin^2 \beta} \end{cases}$$

The angle  $\beta$  is the common MSSM parameter defined at tree level by  $\tan \beta = v_2/v_1$  where  $v_{1,2}$  are the vevs of the two Higgs doublets.

For sfermions (sleptons or squarks) the gauge part does not change. Also the Yukawa terms are identical (matching chiralities). This is a non trivial consequence of supersymmetry and of the general fact that breaking tends to be suppressed at high energy.

### SINGLE TOP PRODUCTION AT LHC: SUDAKOV CORRECTIONS

Single top quark production at LHC is a very interesting topic for the search of New Physics effects via radiative corrections. It is also remarkable due to its special feature of being a direct measure of  $|V_{tb}|^2$ . At leading order there are three basic processes that must be considered, i.e. at parton level, the processes  $bu \rightarrow td$ ,  $u\bar{d} \rightarrow t\bar{b}$  and  $bg \rightarrow tW$ . They have quite different features as discussed in details in [4]. Application of the Sudakov technique to the three processes have been discussed in [5]. Here we discuss in some details the associated  $Wt$  production  $bg \rightarrow tW^-$  as a complete application of the previous general discussion.

The two tree level diagrams to be considered are shown in Fig. (1). In the first (a) a bottom quark is exchanged in the  $s$  channel; In the second (b) a top quark is exchanged in the  $u = (p_b - p_W)^2$  channel. To simplify the discussion let us neglect all masses with the exception of  $m_t$ . We neglect the ratio  $m_t/\sqrt{s}$ , but obviously keep  $m_t/M_W$ . Let us denote the helicity amplitude for the process as  $F_{\alpha\alpha'\beta\beta'}$  where the four subscripts denote the helicities of the four external states  $b$ ,  $g$ ,  $t$ , and  $W$ . In our approximations we remain with the two non suppressed helicity amplitudes

$$F_{----}^{\text{Born a+b}} \rightarrow g_{WL} g_s \left( \frac{\lambda^l}{2} \right) \frac{2}{\cos \frac{\vartheta}{2}}$$

$$F_{-+-+}^{\text{Born a+b}} \rightarrow g_{WL} g_s \left( \frac{\lambda^l}{2} \right) 2 \cos \frac{\vartheta}{2}$$

$$F_{-++0}^{\text{Born a+b}} \rightarrow g_{WL} g_s \left( \frac{\lambda^l}{2} \right) \sqrt{2} \frac{m_t}{M_W} \cos \frac{\vartheta}{2} \frac{1 - \cos \vartheta}{1 + \cos \vartheta}$$

The differential cross section is simply ( $s, t, u$  are standard Mandelstam variables)

$$\frac{d\sigma^{\text{Born}}}{d \cos \vartheta} \rightarrow - \frac{\pi \alpha \alpha_s}{24 s_W^2 u s^2} \left[ s^2 + u^2 + \frac{m_t^2 v^2}{2 M_W^2} \right]$$

We can now write in a very simple way the Sudakov electroweak corrections to these two non suppressed amplitudes.

The universal component for producing a transverse  $W$  is

$$\frac{F_{-,\mu,-,\mu}^{\text{Univ}}}{F_{-,\mu,-,\mu}^{\text{Born}}} = \frac{1}{2} ( c^{\text{ew}}(b\bar{b})_L + c^{\text{ew}}(t\bar{t})_L ) + c^{\text{ew}}(W_T)$$

where  $c^{\text{ew}}(f\bar{f})$  is the sum of the gauge and Yukawa correction associated to a  $f$  fermion line. We have also introduced the new coefficient

$$c^{\text{ew}}(W_T) = \frac{\alpha}{4\pi s_W^2} \left( -\log^2 \frac{s}{M_W^2} \right)$$

In the production of longitudinal  $W_0^-$  we have instead

$$\frac{F_{-,+,+0}^{\text{Univ}}}{F_{-,+,+0}^{\text{Born}}} = \frac{1}{2} ( c^{\text{ew}}(b\bar{b})_L + c^{\text{ew}}(t\bar{t})_R ) + c^{\text{ew}}(W_0)$$

where

$$c^{\text{ew}}(W_0) = \frac{\alpha}{\pi} \left( \frac{1+2c_W^2}{32 s_W^2 c_W^2} \right) \left( n_G \log \frac{s}{M_W^2} - \log^2 \frac{s}{M_W^2} \right)$$

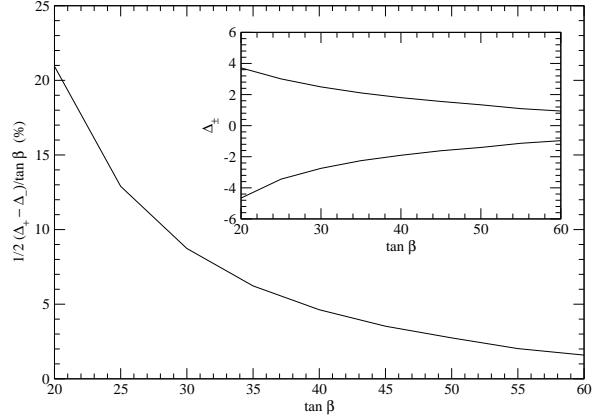
and  $n_G = 4, 0$  in the Standard Model or MSSM, respectively.

For reasons of space, we do not write the explicit angular dependent terms. Finally there are SUSY QCD Sudakov logarithms from vertices with gluino exchanges. They read

$$F_{-,\mu,-,\mu}^{\text{Univ SUSYQCD}} = F_{-,\mu,-,\mu}^{\text{Born}} \left( -\frac{\alpha_s}{3\pi} \log \frac{s}{M_{\text{SUSY}}^2} \right)$$

$$F_{-,+,+0}^{\text{Univ SUSYQCD}} = F_{-,+,+0}^{\text{Born}} \left( -\frac{\alpha_s}{3\pi} \log \frac{s}{M_{\text{SUSY}}^2} \right)$$

In this specific process there are no RG logarithms.



**FIGURE 2.** Bounds on  $\tan \beta$  from the combination of the three single top production processes.

## NUMERICAL RESULTS

The same set of corrections can be computed for the other three processes of single top production. The partonic cross sections are then converted in hadronic observables. In particular, one defines the distribution

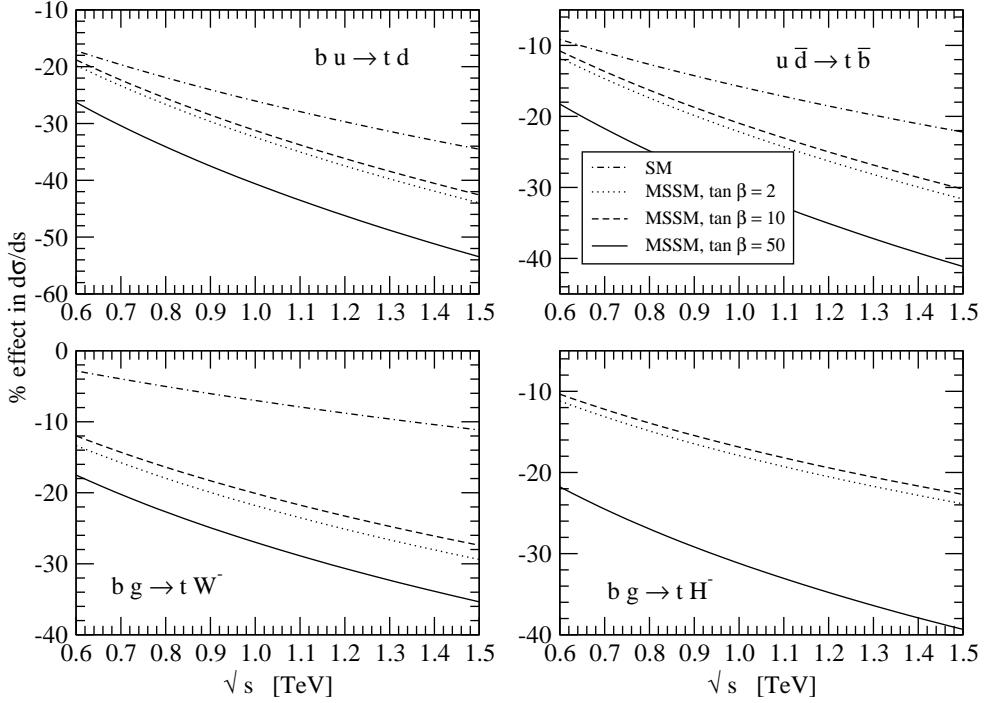
$$\frac{d\sigma(PP \rightarrow tY + \dots)}{ds} = \frac{1}{\int_{-\cos \bar{\vartheta}}^{\cos \bar{\vartheta}} d\cos \vartheta} \sum_{ij} L_{ij}(\tau, \cos \vartheta) \frac{d\sigma_{ij \rightarrow tY}}{d\cos \vartheta}(s)$$

where  $\sqrt{S}$  is the  $pp$  energy,  $L_{ij}$  is the luminosity from partons  $i, j$  and the various kinematical variables are explained in details in [5].

The results for the three single top production processes are summarized in Fig. (3). Large effects can be obtained due to the Yukawa terms. These effects isolate the parameter  $\tan \beta$  and can be used to fix bounds on it. To give an example of such a procedure, we have combined the effects in the three production processes assuming measurements in the range  $\sqrt{s} = 500 - 1500$  GeV (with 20 GeV spacing) and an overall precision of 10%. Fig. (2) illustrates the results and show the values of  $\Delta_{\pm}$  which appear in the confidence region  $(\tan \beta + \Delta_{-}, \tan \beta + \Delta_{+})$  which is determined by a  $\chi^2$  analysis of fake data simulated at a certain  $\tan \beta$ .

In practice, the simple Sudakov approximation displays interesting features and suggest that a full one loop calculation would be certainly worthwhile.

A similar analysis in the case of top - antitop pair production has also been completed and can be found in [6].



**FIGURE 3.** Effects in  $d\sigma/ds$  for the three single top production processes.

## REFERENCES

1. M. Kuroda, G. Moultsaka and D. Schildknecht, Nucl. Phys. **B 350**, 25 (1991); A. Denner, S. Dittmaier and R. Schuster, Nucl. Phys. **B 452**, 80 (1995); A. Denner, S. Dittmaier and T. Hahn, Phys. Rev. **D 56**, 117 (1997); A. Denner and T. Hahn, Nucl. Phys. **B 525**, 27 (1998); M. Beccaria, G. Montagna, F. Piccinini, F. M. Renard and C. Verzegnassi, Phys. Rev. **D 58**, 093014 (1998); P. Ciafaloni and D. Comelli, Phys. Lett. **B 446**, 278 (1999).
2. M. Beccaria, S. Prelovsek, F. M. Renard and C. Verzegnassi, Phys. Rev. D **64**, 053016 (2001) [arXiv:hep-ph/0104245]. M. Beccaria, F. M. Renard, S. Trimarchi and C. Verzegnassi, Phys. Rev. D **68**, 035014 (2003) [arXiv:hep-ph/0212167].
3. M. Beccaria, M. Melles, F. M. Renard, S. Trimarchi and C. Verzegnassi, Int. J. Mod. Phys. A **18**, 5069 (2003) [arXiv:hep-ph/0304110].
4. M. Beneke *et al.*, arXiv:hep-ph/0003033.
5. M. Beccaria, F. M. Renard and C. Verzegnassi, Phys. Rev. D **71**, 033005 (2005) [arXiv:hep-ph/0410089].
6. M. Beccaria, S. Bentvelsen, M. Cobal, F. M. Renard and C. Verzegnassi, Phys. Rev. D **71**, 073003 (2005) [arXiv:hep-ph/0412249].